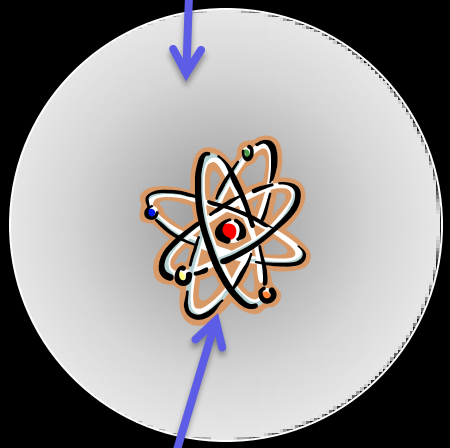
Spinor Bose gases

Dan Stamper-Kurn
UC Berkeley

1. **Some interesting phenomena in multi-component Bose gases**: Fragmentation, symmetry breaking, magnetism
2. **Spinor gas**: Definition, symmetries and interactions, mean-field and many-body ground states
3. **Experimental realities**: spin conservation, first studies
4. **Detecting internal-state coherence optically**
5. **Spin dynamics**: oscillations, quantum quench, dynamical instability
6. **Future directions**

Spinor gases

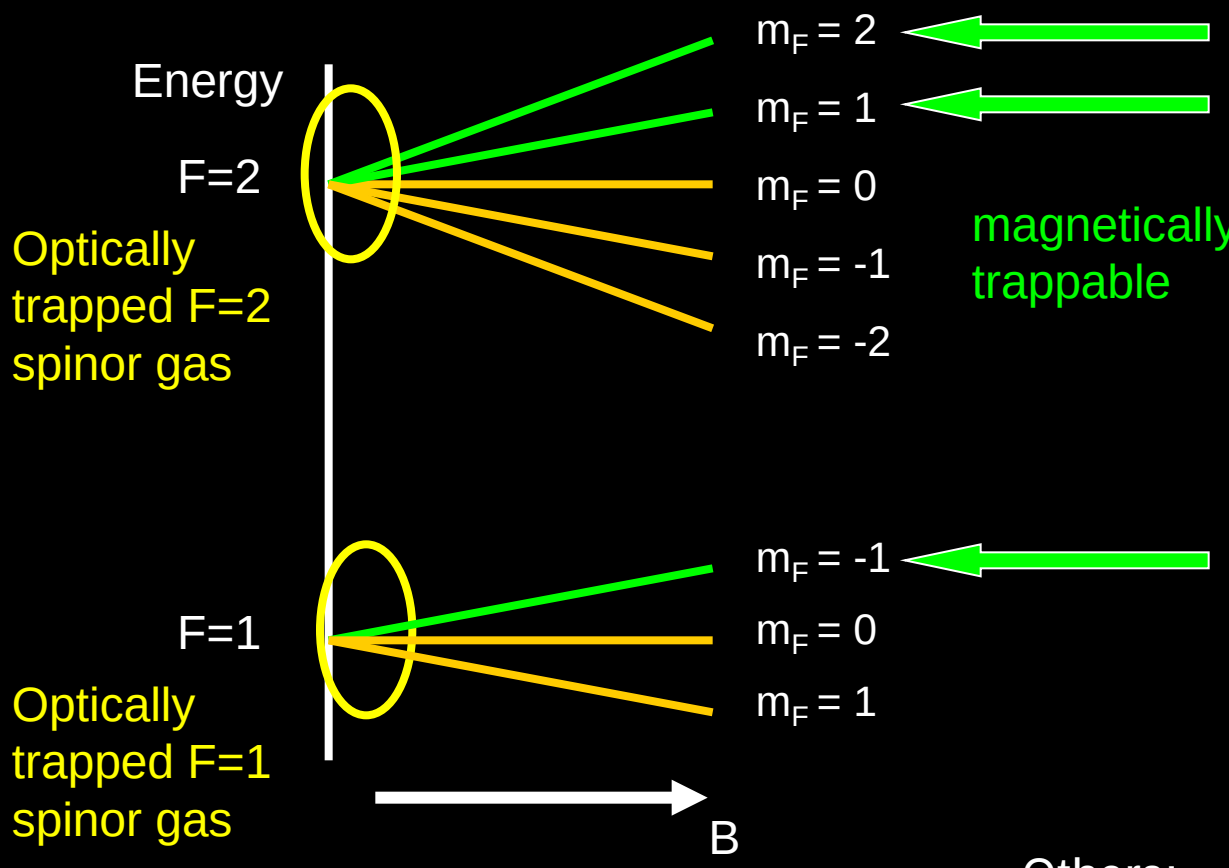
Example: 87-Rb



37 electrons
J = 1/2

37 protons
50 neutrons
I = 3/2

$$\vec{F} = \vec{I} + \vec{J}$$



Others:

- ^{23}Na, F=1
- ^{52}Cr, F=3

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   visualizing states: Majorana, Nuevo Cimento, **9**, 43 (1932); Bloch, Rabi, Rev. Mod. Phys. **17**, 237 (1945); Barnett, Turner, Demler PRL **97**, 180412 (2006).
   inert states: Makela and Suominen, PRL **99**, 190408 (2007), Yip, PRA **75**, 023625 (2007)
   many-body: Law, Pu, Bigelow, PRL **81**, 5257 (1998)
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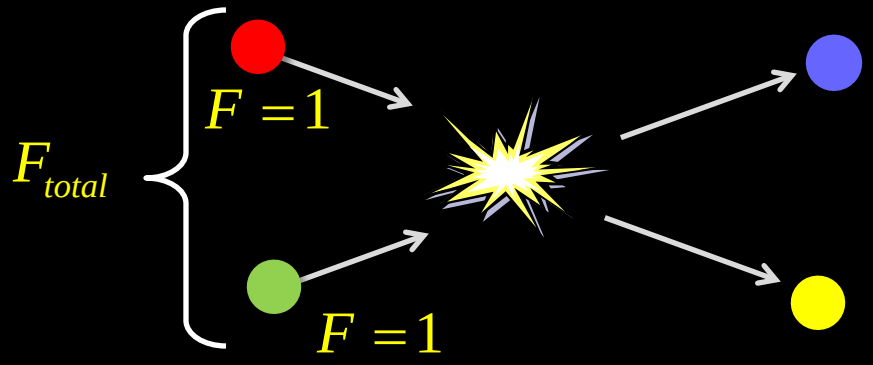
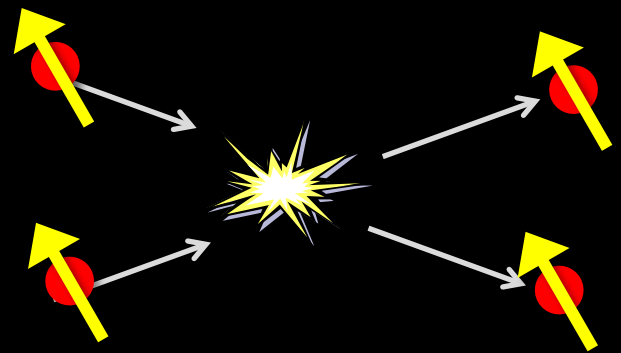
Interatomic interactions



- Low energy
  - only s-wave collisions occur
  - interactions characterized by scattering length
- Rotational symmetry: interactions depend on total spin, not its orientation

	$F_{total} = 0$	$F_{total} = 2$
^{87}Rb:	a_0 = 5.39 nm	a_2 = 5.31 nm

interactions are repulsive



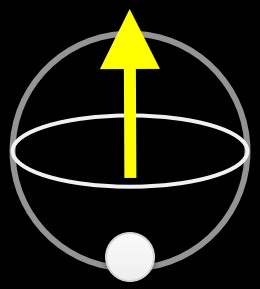
slightly less repulsive
“ferromagnetic”

Quantum states of an F=1 atom

Examples:

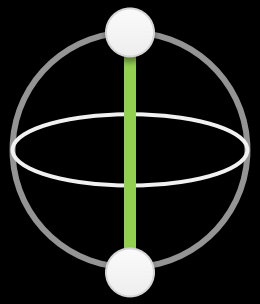
"magnetic"
"oriented"

$$\Psi = \hat{R}\left| m_z = 1 \right\rangle$$



"non-magnetic"
"nematic"
"aligned"

$$\Psi = \hat{R}\left| m_z = 0 \right\rangle$$



Barnett, Turner, Demler
[PRL **97**, 180401 (2007)]

F=2

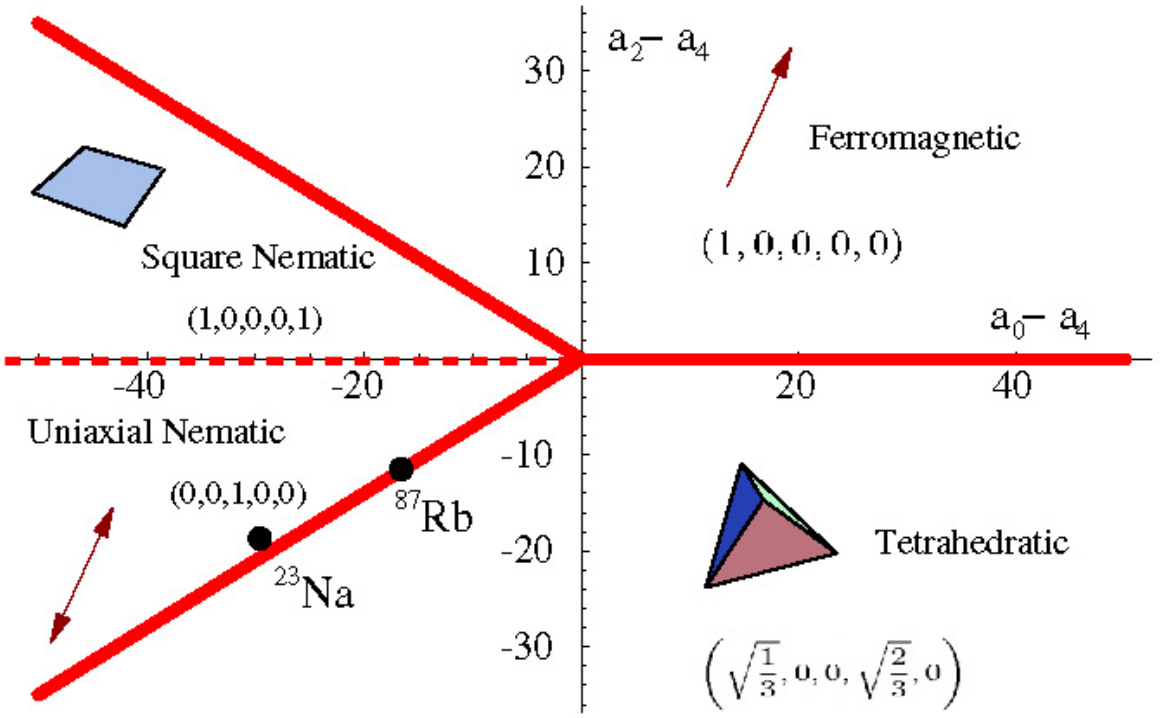


FIG. 1 (color online). The phase diagram for a spin-2 spinor condensate. The dashed line corresponds to the additional phase boundary dividing the nematic phase into two phases, a uniaxial nematic phase and a square nematic phase.

- resolution of nematic states through “order by disorder” (pK energy scales, probably inaccessible)
  Turner, Barnett, Demler, Vishwananth, PRL **98**, 190404 (2007)

and higher...

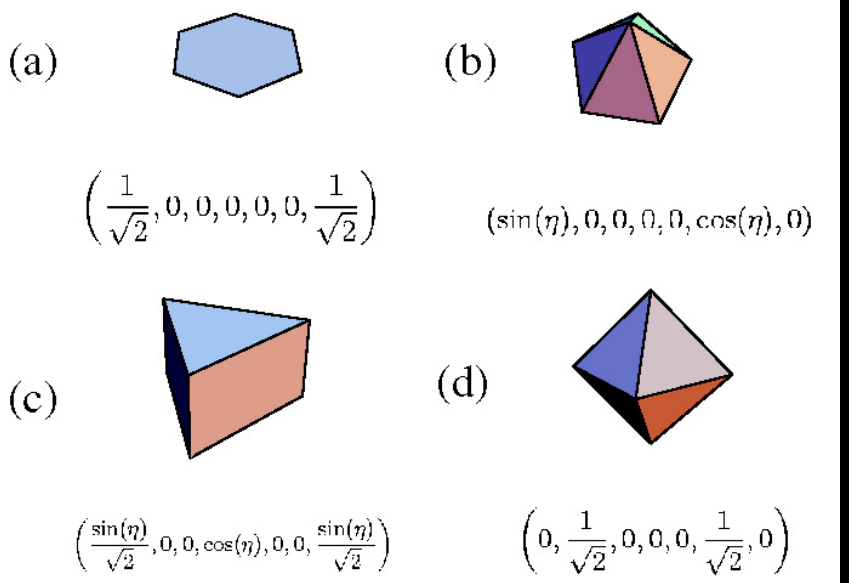


FIG. 2 (color online). Some possible phases that can be realized for a spin-three system either in the superfluid or Mott insulating states. Shown are the coefficients of the wave functions A_α and the shapes representing the symmetries of the wave functions. The phases transform as the following polyhedra: (a) the hexagon, (b) the pyramid with pentagonal base, (c) the prism, and (d) the octahedron. For phase (c) we have the condition $\tan^2(\eta) < 10$. Additional phases similar to the spin-two case (not shown) are the ferromagnetic and nematic states.

Turner, Barnett, Demler

TABLE I. The inert states for $S = 1$–4. The only symmetry group that is not manifested here is the icosahedral group.

Spin	Inert states
$S = 1$	$\xi_{SO(2)} = \|1, 1\rangle$, $\xi_{O(2)} = \|1, 0\rangle$
$S = 2$	$\xi_{SO(2)} = \|2, 2\rangle$ and $\|2, 1\rangle$, $\xi_{D_4} = \|2, 2\rangle + \|2, -2\rangle$, $\xi_{O(2)} = \|2, 0\rangle$, and $\xi_{\text{Tetra}} = \|2, 2\rangle + i\sqrt{2}\|2, 0\rangle + \|2, -2\rangle$
$S = 3$	$\xi_{SO(2)} = \|3, 3\rangle$, $\|3, 2\rangle$, and $\|3, 1\rangle$, $\xi_{O(2)} = \|3, 0\rangle$, $\xi_{D_6} = \|3, 3\rangle + \|3, -3\rangle$, and $\xi_{\text{Octa}} = \|3, 2\rangle + \|3, -2\rangle$
$S = 4$	$\xi_{SO(2)} = \|4, 4\rangle$, $\|4, 3\rangle$, $\|4, 2\rangle$, and $\|4, 1\rangle$, $\xi_{O(2)} = \|4, 0\rangle$, $\xi_{D_8} = \|4, 4\rangle + \|4, -4\rangle$, $\xi_{D_6} = \|4, 3\rangle + \|4, -3\rangle$ $\xi_{D_4} = \|4, 2\rangle + \|4, -2\rangle$, $\xi_{\text{Tetra}} = \sqrt{7}\|4, 4\rangle + 2i\sqrt{3}\|4, 2\rangle$ $-\sqrt{10}\|4, 0\rangle + 2i\sqrt{3}\|4, -2\rangle + \sqrt{7}\|4, -4\rangle$. $\xi_{\text{Cube}} = \sqrt{5}\|4, 4\rangle - \sqrt{14}\|4, 0\rangle + \sqrt{5}\|4, -4\rangle$

Makela, Suominen

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Energy scales in a spinor Bose-Einstein condensate

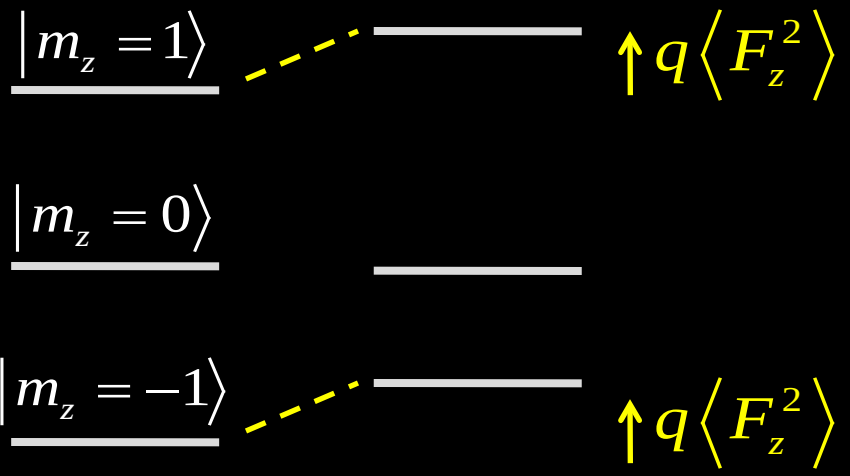
- spin-independent contact interactions

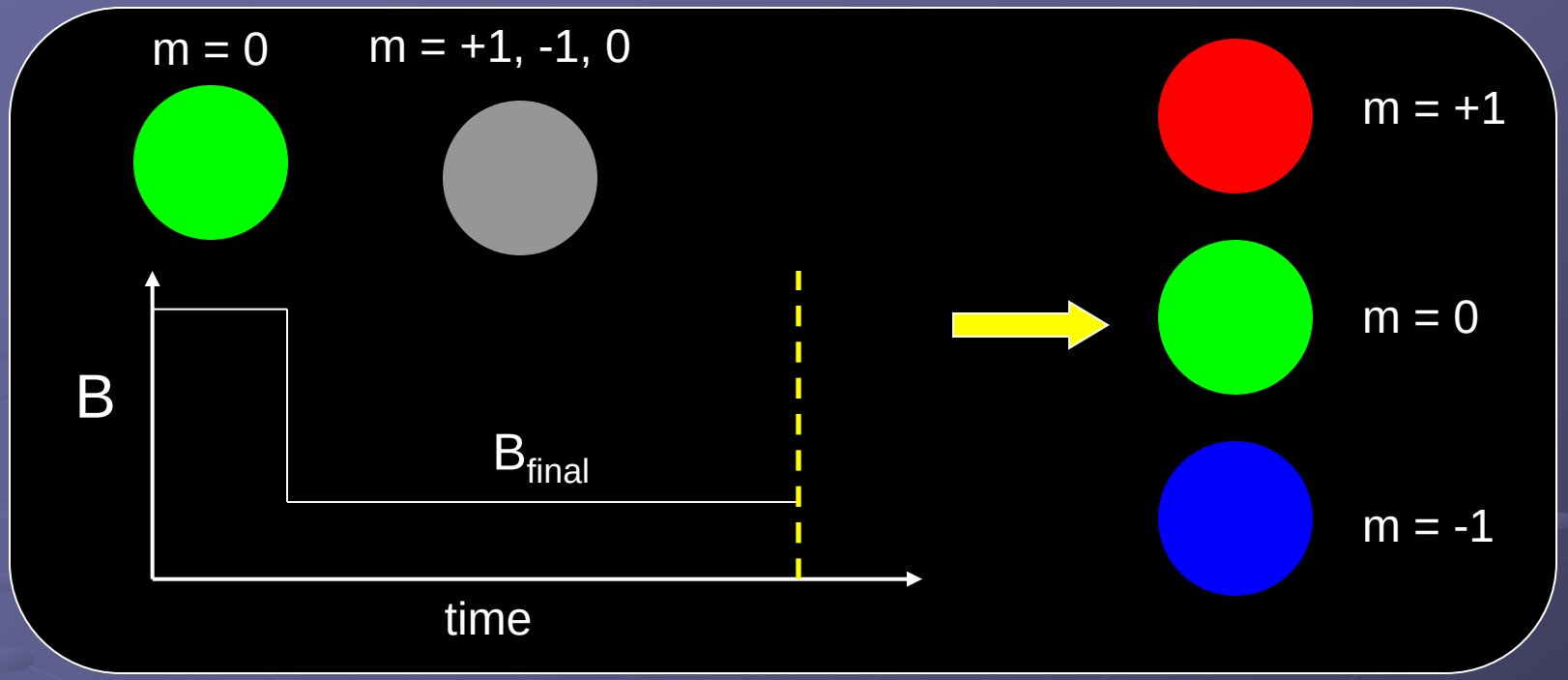
$$\mu = c_0 n$$ ≈ 2000 Hz, or 100 nK

- spin-dependent contact interactions

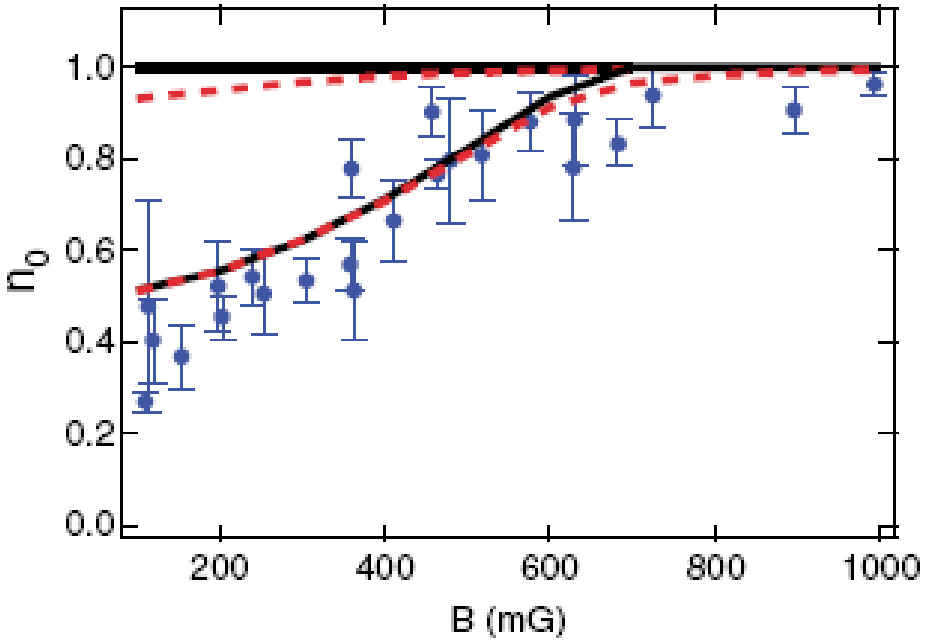
$$\Delta\mu = -|c_2| n \left\langle \vec{F} \right\rangle^2$$ ≈ 10 Hz, or 0.5 nK

- quadratic Zeeman shift





F=1 spinor BEC of ^{87}Rb



Chapman et al., PRL **92**,140403 (2004);
Sengstock et al., PRL **92**,040402 (2004)

$$\vec{\Psi} = \begin{pmatrix} \frac{1}{2} e^{i\phi_1} \\ \frac{1}{\sqrt{2}} e^{i\phi_0} \\ \frac{1}{2} e^{i\phi_{-1}} \end{pmatrix}$$

?

$$\vec{\Psi} = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{\sqrt{2}} \\ \frac{1}{2} \end{pmatrix}$$

Ferromagnetic state

$$\left\langle \vec{F} \right\rangle = 1$$

points in x-direction

?

$$\vec{\Psi} = \begin{pmatrix} \frac{1}{2} \\ i\frac{1}{\sqrt{2}} \\ \frac{1}{2} \end{pmatrix}$$

Polar state !!!

$$\left\langle \vec{F} \right\rangle = 0$$

“points nowhere”
along the y+z axis

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What do we want to know?

$$\rho = \begin{pmatrix} \rho_{+1,+1} & \rho_{+1,0} & \rho_{+1,-1} \\ \rho_{0,+1} & \rho_{0,0} & \rho_{0,-1} \\ \rho_{-1,+1} & \rho_{-1,0} & \rho_{-1,-1} \end{pmatrix}$$

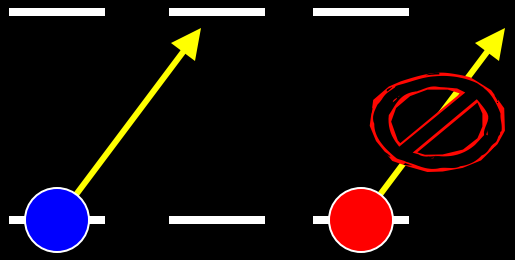
$$\chi = \begin{pmatrix} \chi_{\sigma^+,\sigma^+} & \chi_{\sigma^+,\pi} & \chi_{\sigma^+,\sigma^-} \\ \chi_{\pi,\sigma^+} & \chi_{\pi,\pi} & \chi_{\pi,\sigma^-} \\ \chi_{\sigma^-,\sigma^+} & \chi_{\sigma^-,\pi} & \chi_{\sigma^-,\sigma^-} \end{pmatrix}$$

spin F
Δm=1 coherences
“orientation”

nematicity N
Δm=2 coherences
“alignment”

(Full) optical characterization of spinor gas

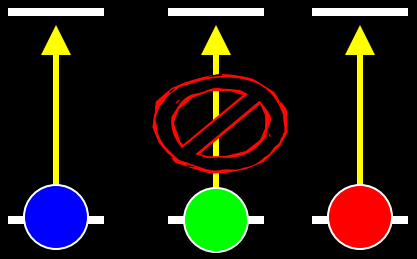
probe with σ^+ light



m_F=-1 m_F=0 m_F=1

Circular birefringence: reveals magnetization (spin)

probe with π light

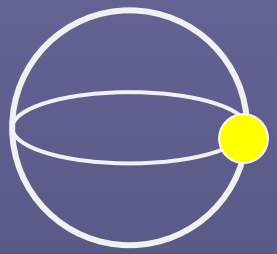


m_F=-1 m_F=0 m_F=1

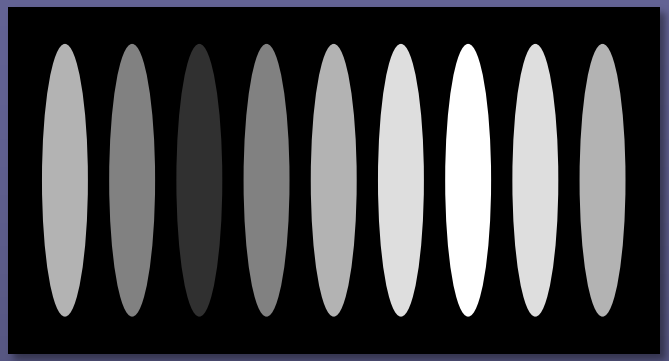
Linear birefringence: reveals alignment (nematicity)

Measuring the vector spin

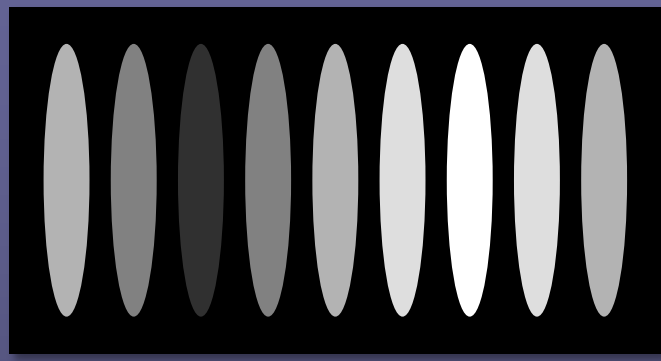
- Larmor precession: continuous spin rotation about z-axis
- resonant RF pulses: a $\pi/2$ spin rotation about x-axis

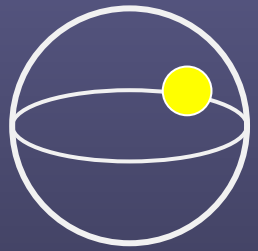


t=0 spin along x

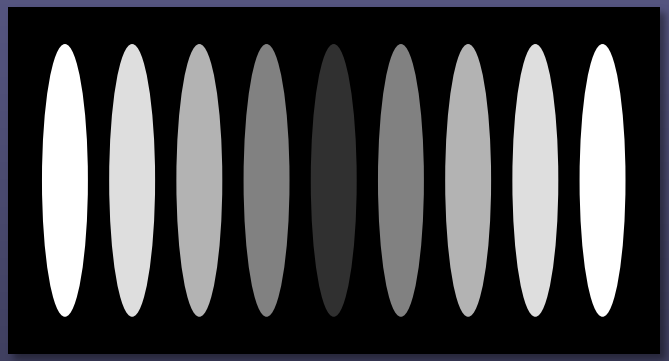


$\pi/2$

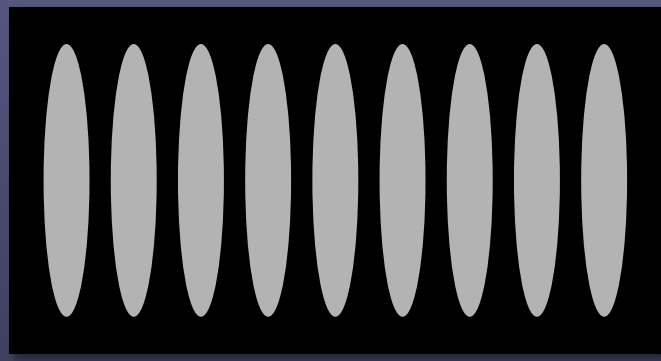


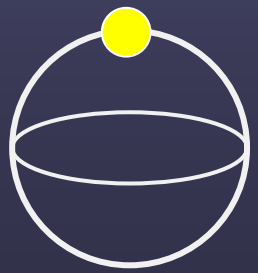


t=0 spin along y

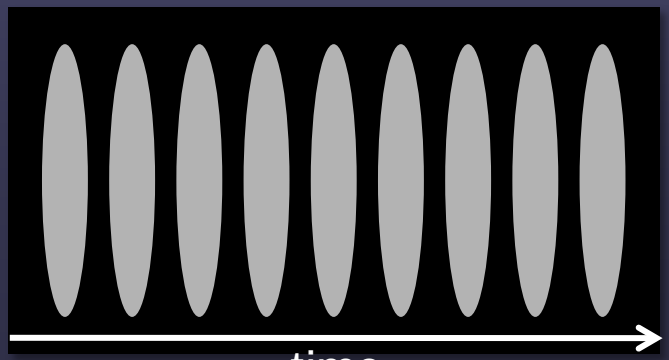


$\pi/2$

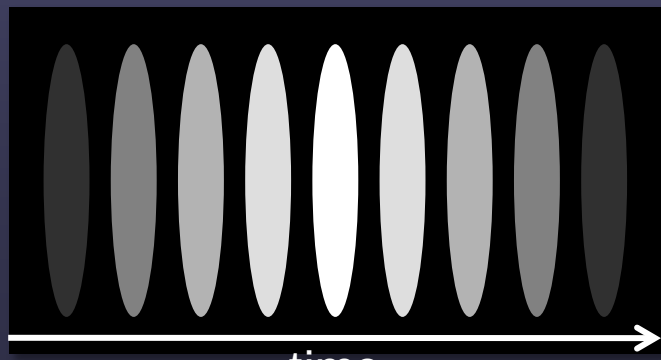


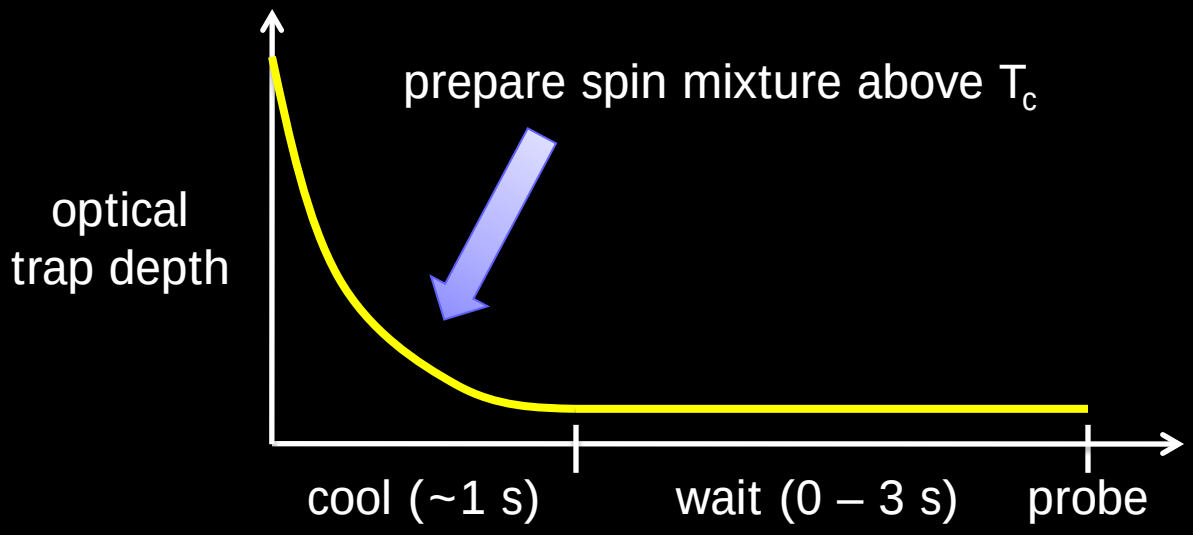


t=0 spin along z

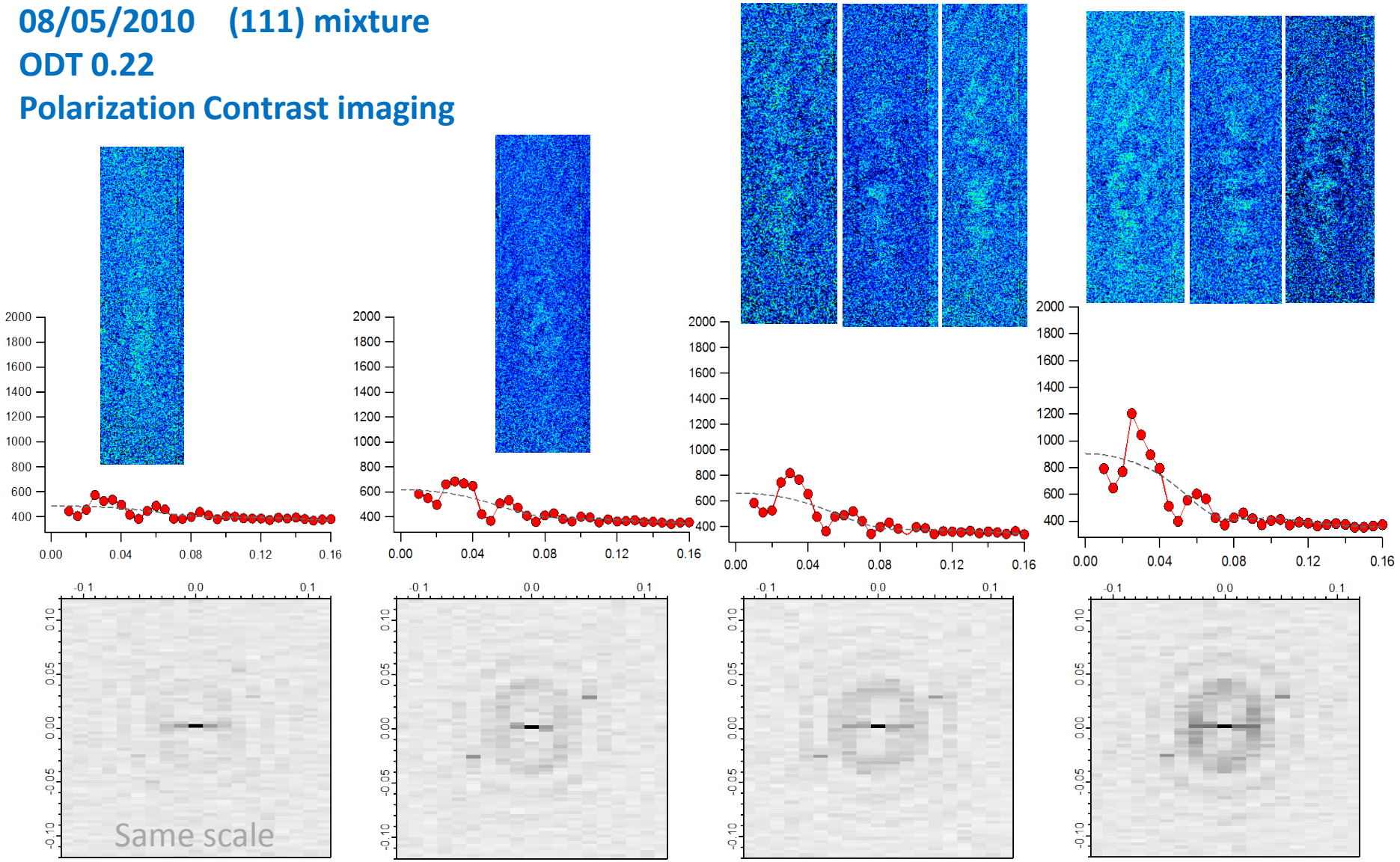


$\pi/2$




prepare spin mixture above T_c
optical
trap depth
cool (~1 s)
wait (0 – 3 s)
probe

08/05/2010 (111) mixture
ODT 0.22
Polarization Contrast imaging

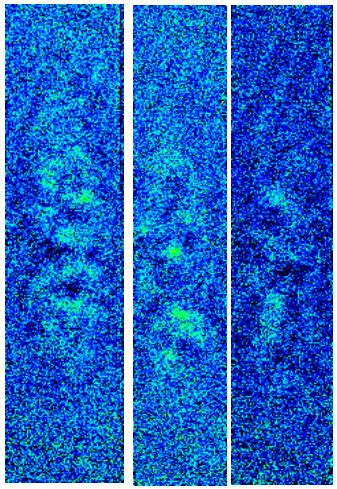
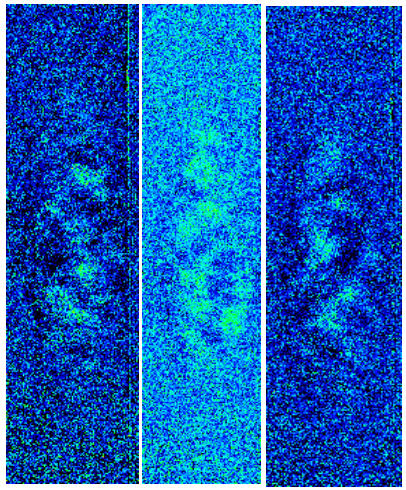
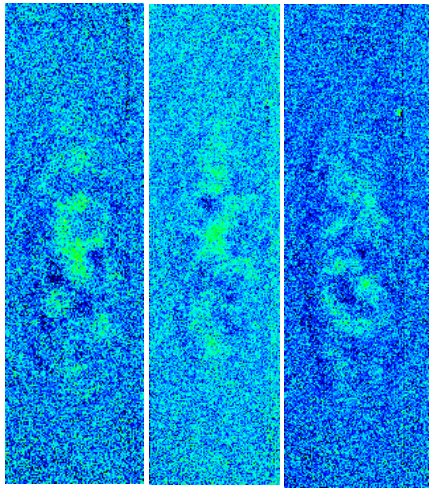
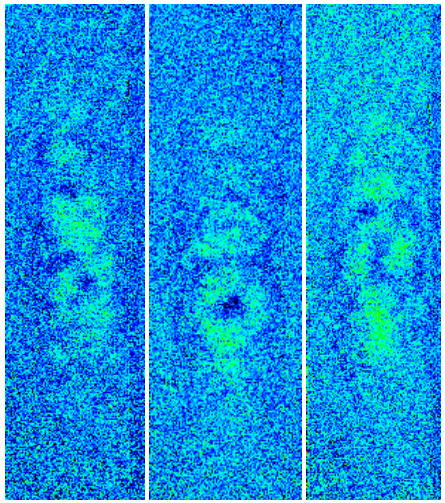
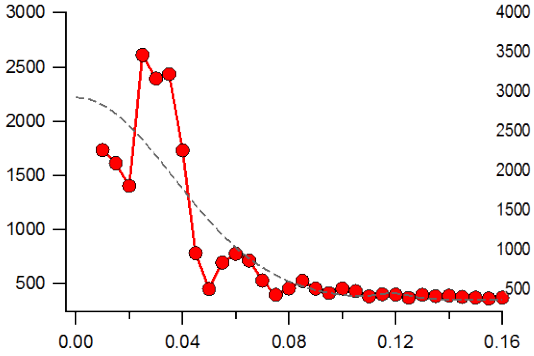


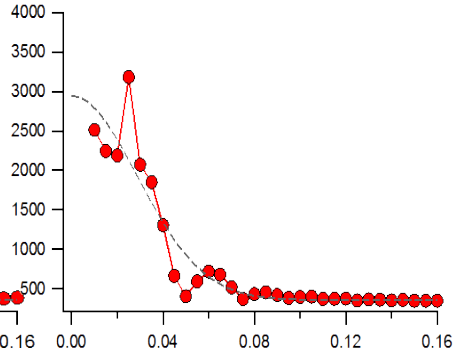
(111) Mixture @ 50ms

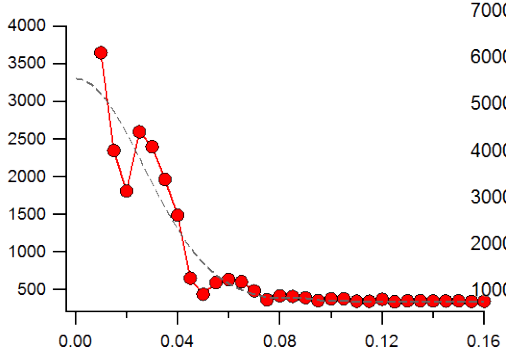
(111) Mixture @ 100ms

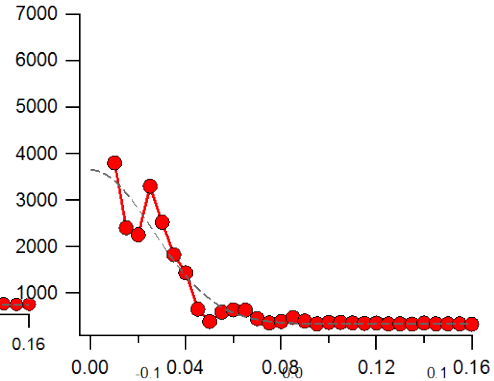
(111) Mixture @ 150ms

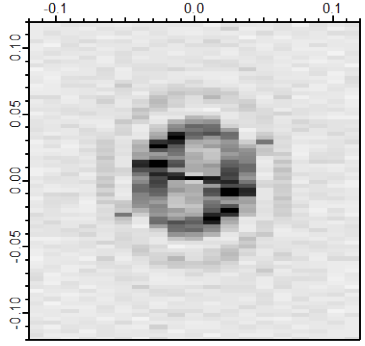
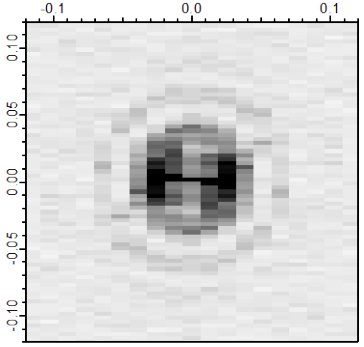
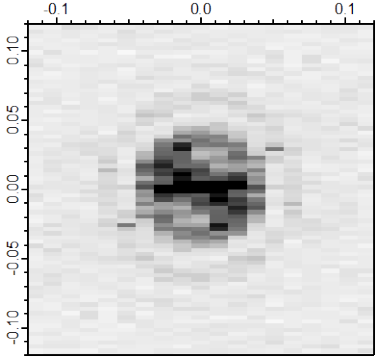
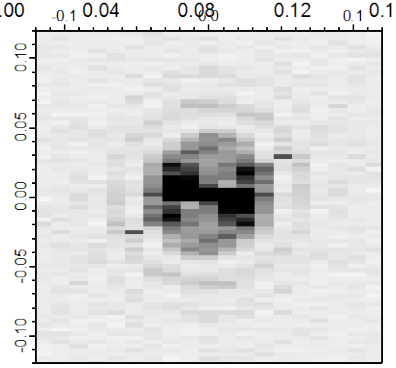
(111) Mixture @ 200ms







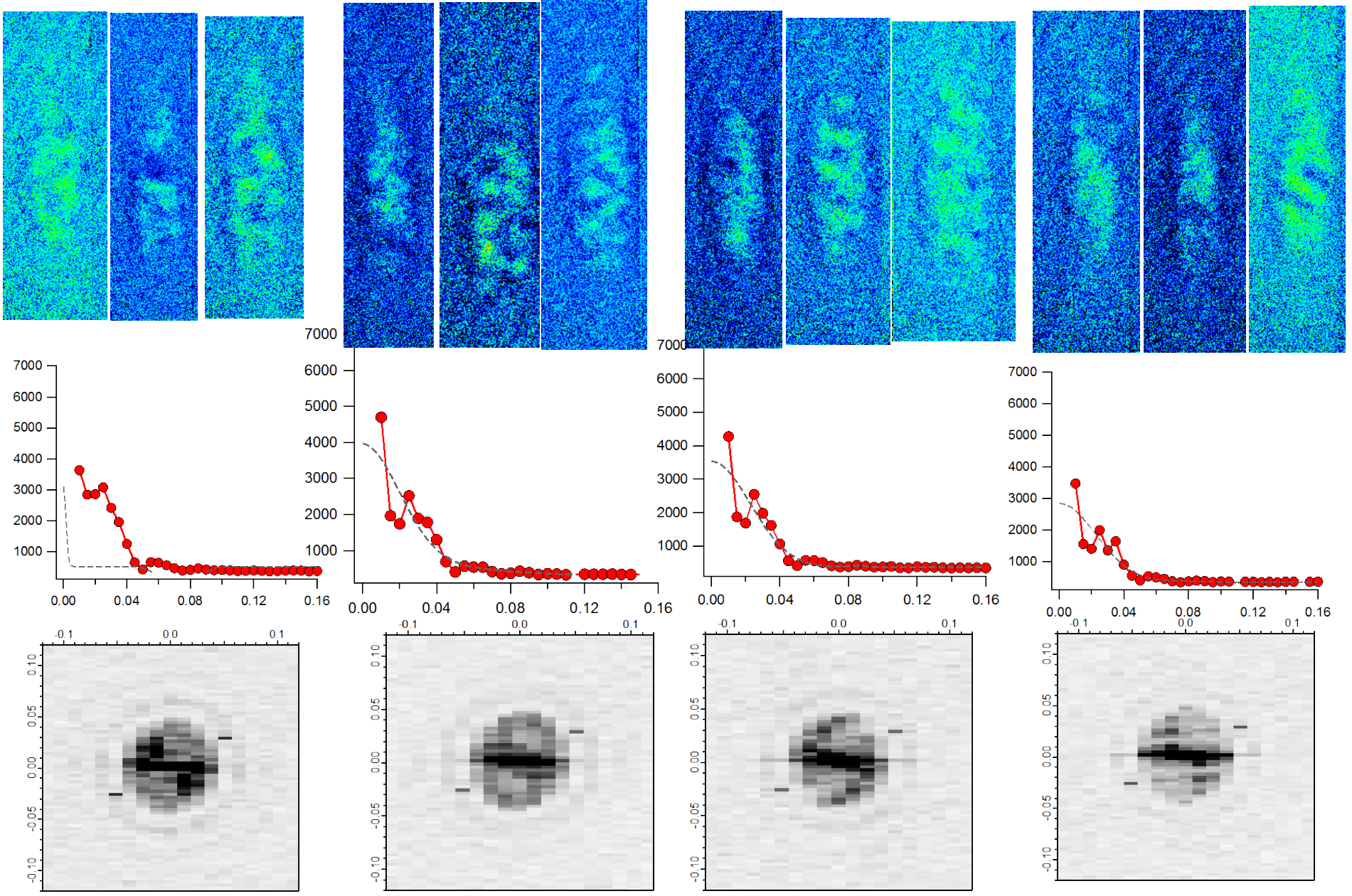





(111) Mixture @ 500ms

(111) Mixture @ 700ms

(111) Mixture @ 1000ms

(111) Mixture @ 1200ms



(111) Mixture @ 1350ms

(111) Mixture @ 1500ms

(111) Mixture @ 1600ms

(111) Mixture @ 1800ms

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spin mixing of many atom pairs

Widera et al., PRL **95**, 190405 (2005)

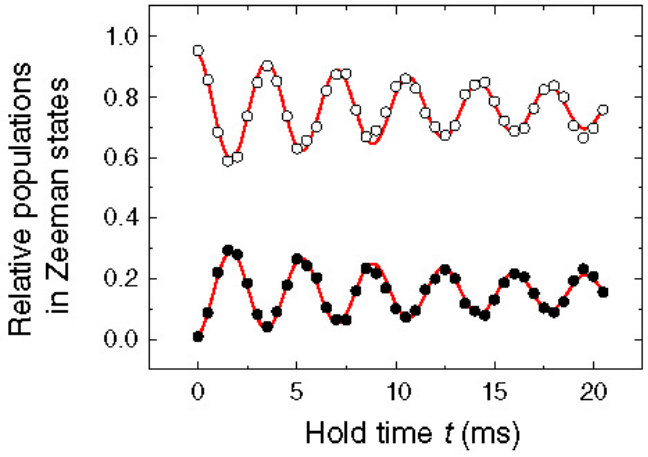
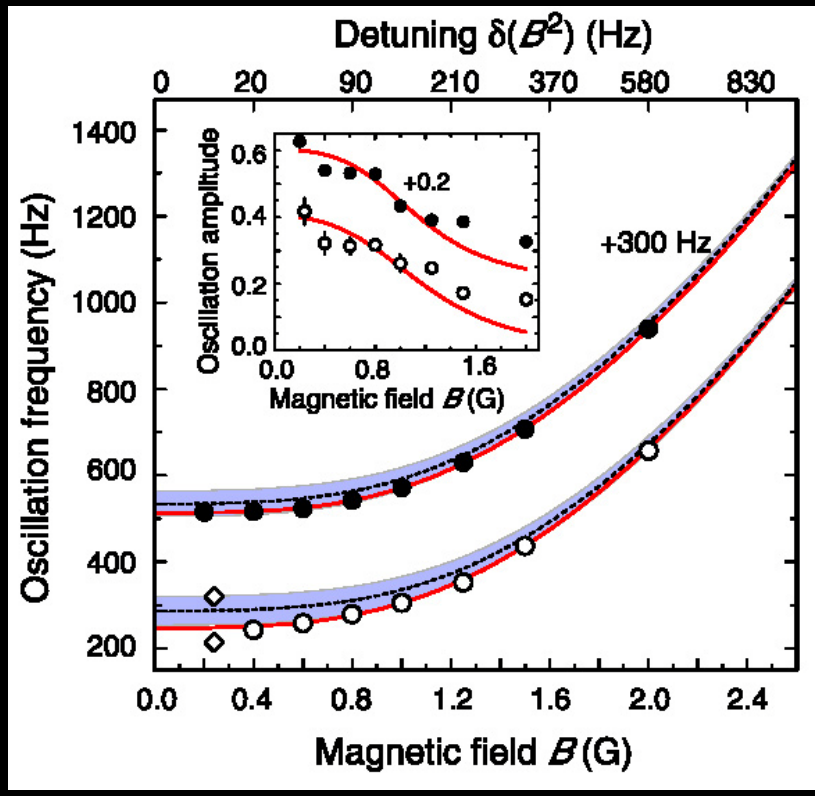


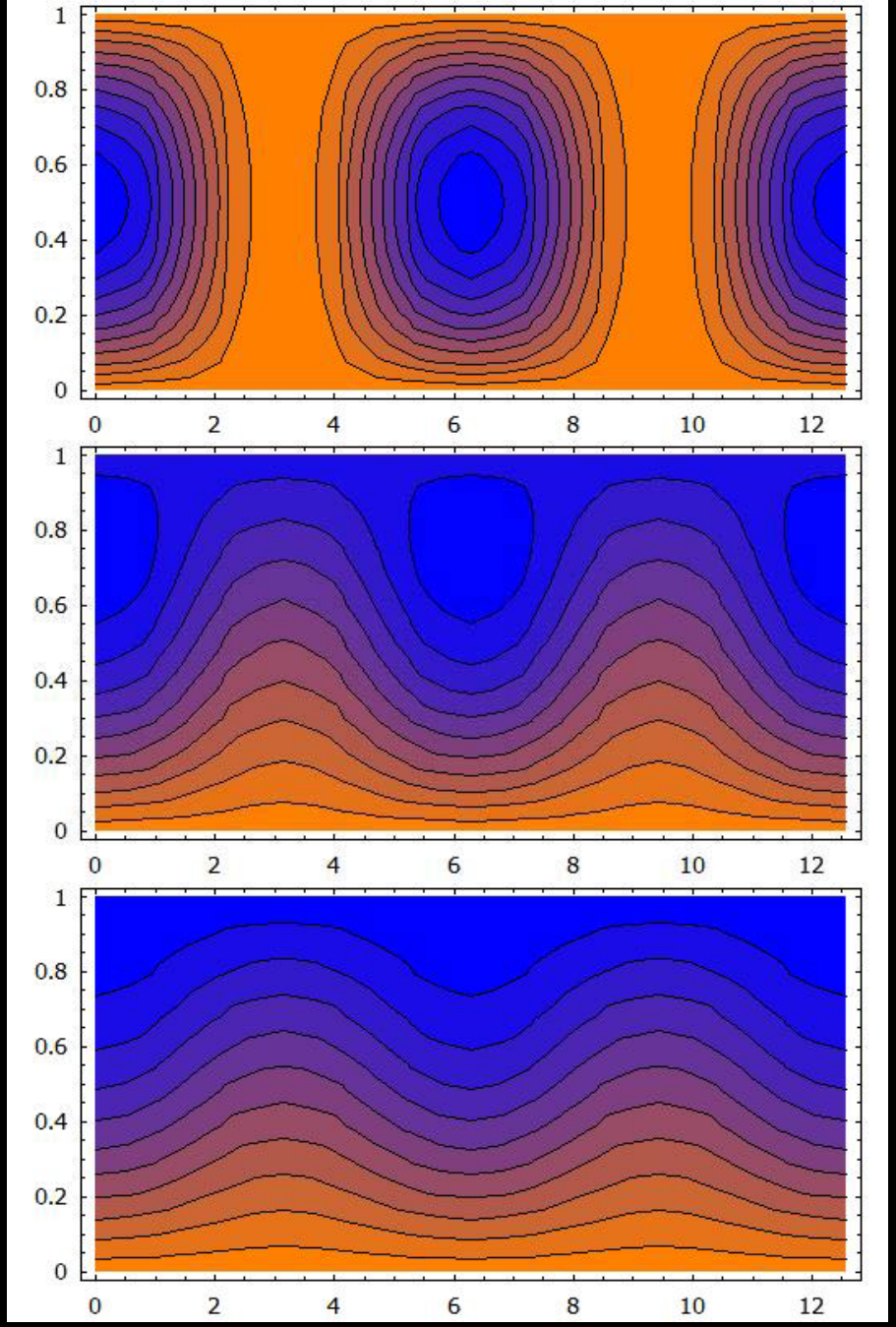
FIG. 2 (color online). Spin dynamics of atom pairs localized in an optical lattice at a magnetic field of $B = 0.8$ G. The atoms are initially prepared in $|0, 0\rangle$ and can evolve into $|+1, -1\rangle$. Shown are the populations in $m_f = 0$ (○) and $m_f = \pm 1$ (●) together with a fit to a damped sine yielding an oscillation frequency of $\Omega'_{if} = 2\pi \times 278(3)$ Hz.



Coherent spin mixing: mean-field dynamics

Zhang, et al., PRA **72**, 013602 (2005)

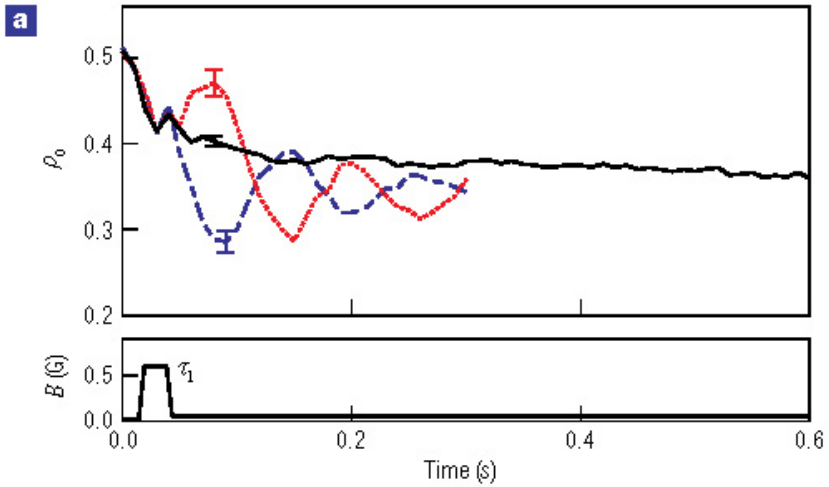
ferromagnetic case shown

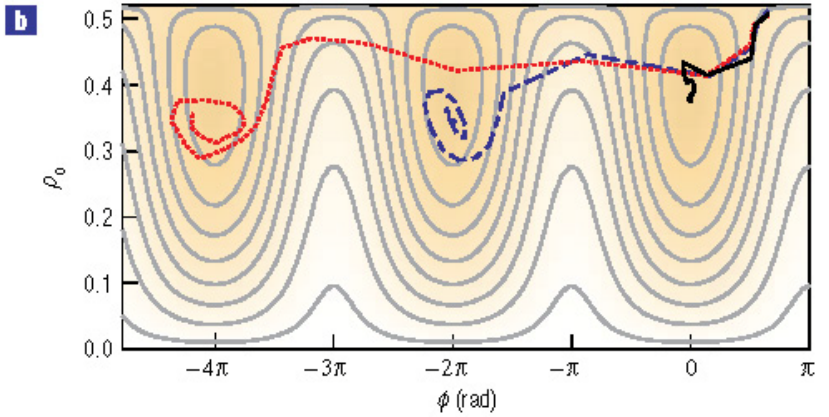


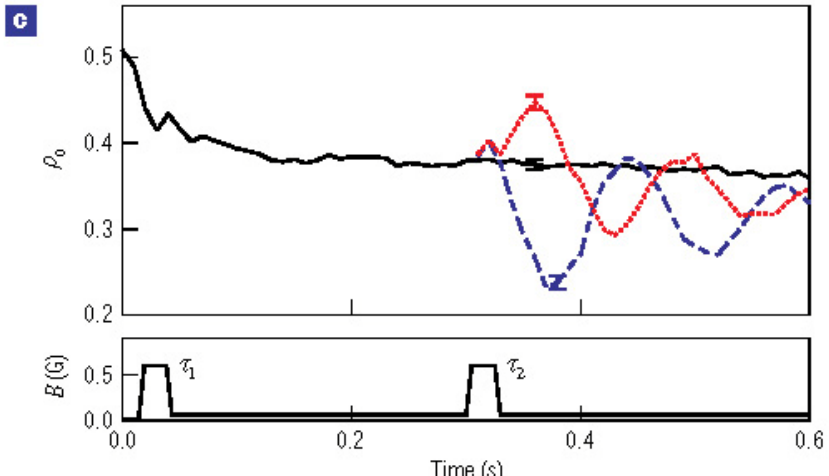
$q = 0$

$q = |c_2|\, n$

$q = 2|c_2|\, n$



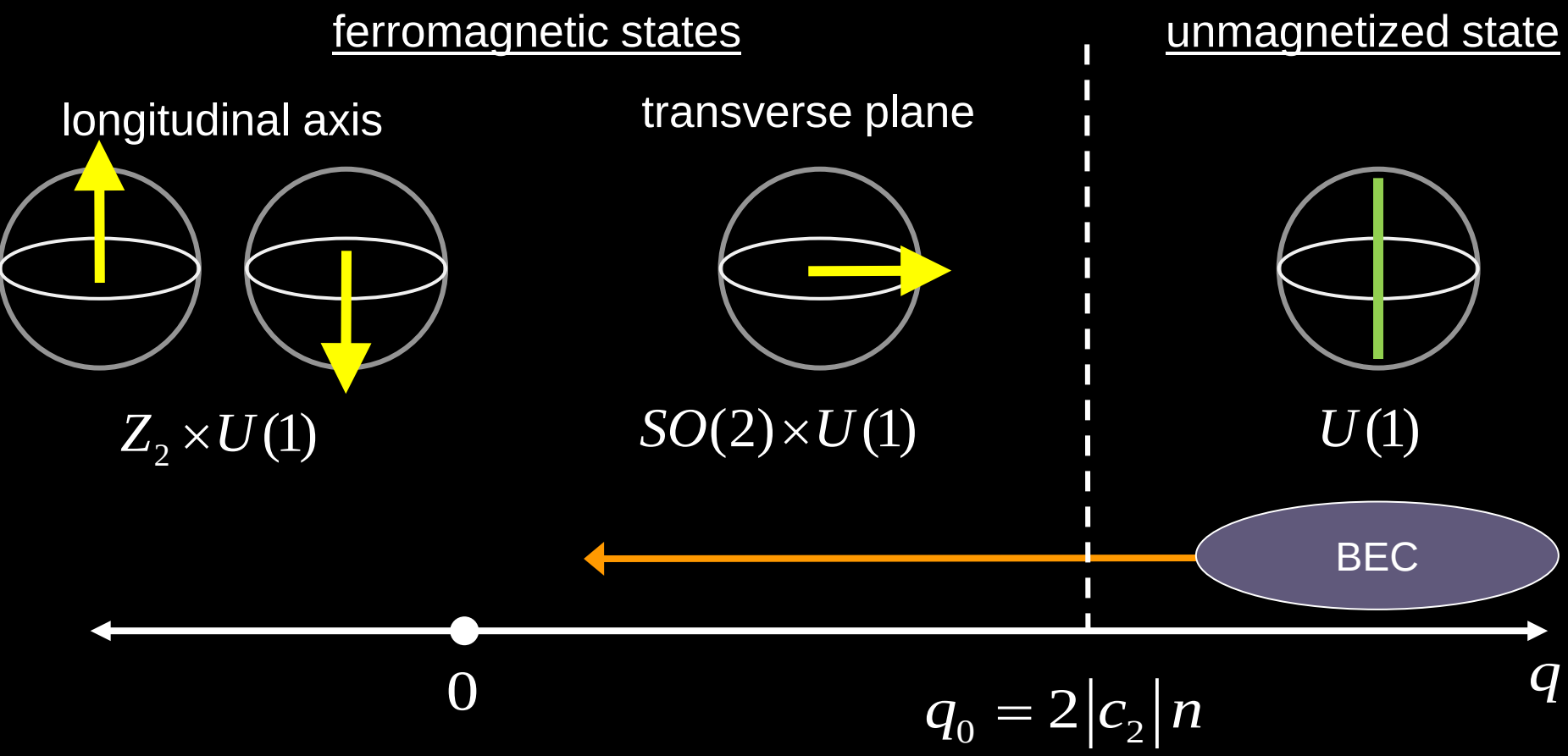




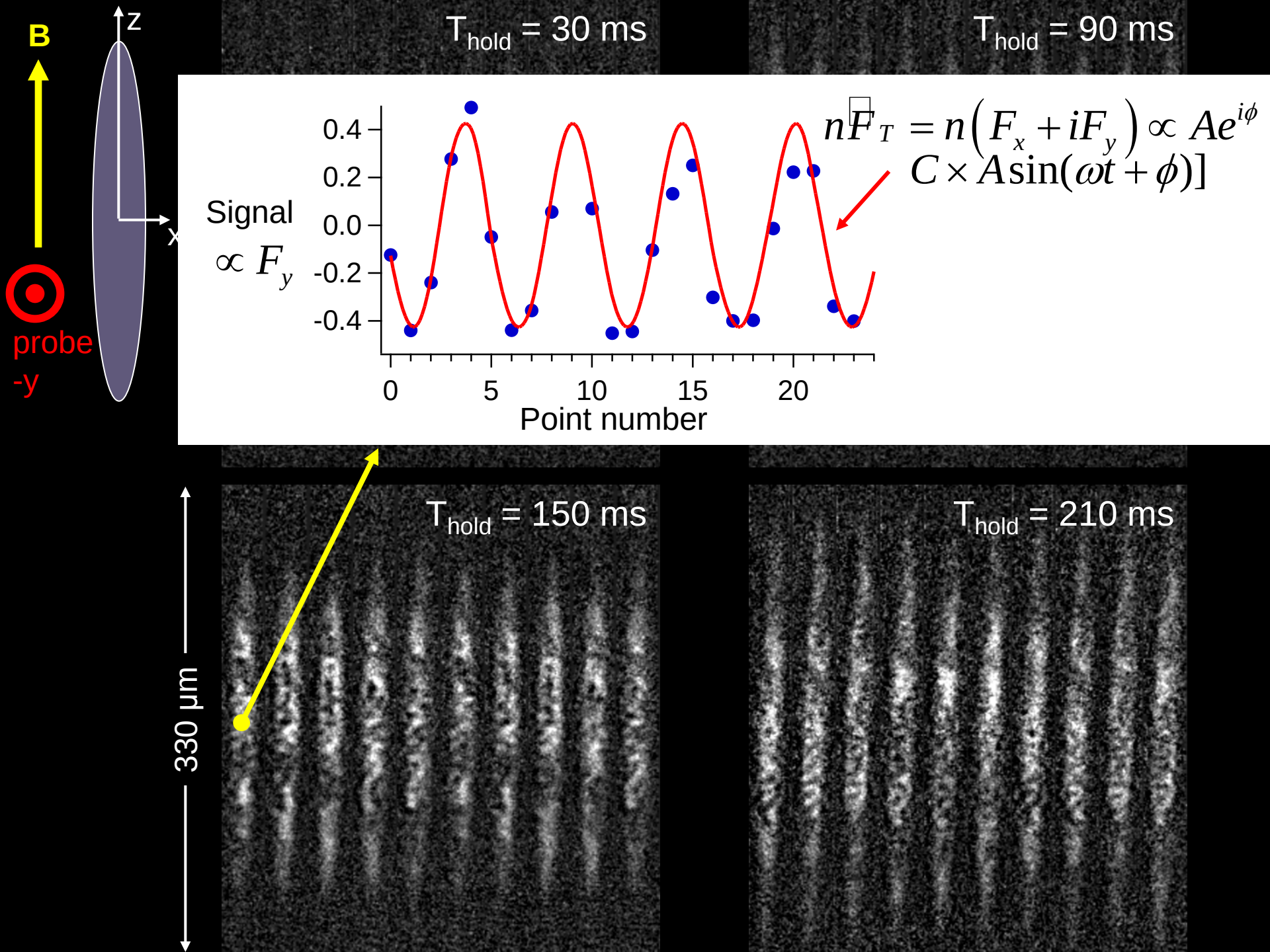
M. S. Chang et al, Nature Physics **1**, 111 (2005)

Phases and symmetries

$$E = -|c_2| n \left\langle \vec{F} \right\rangle^2 + q \left\langle F_z^2 \right\rangle$$

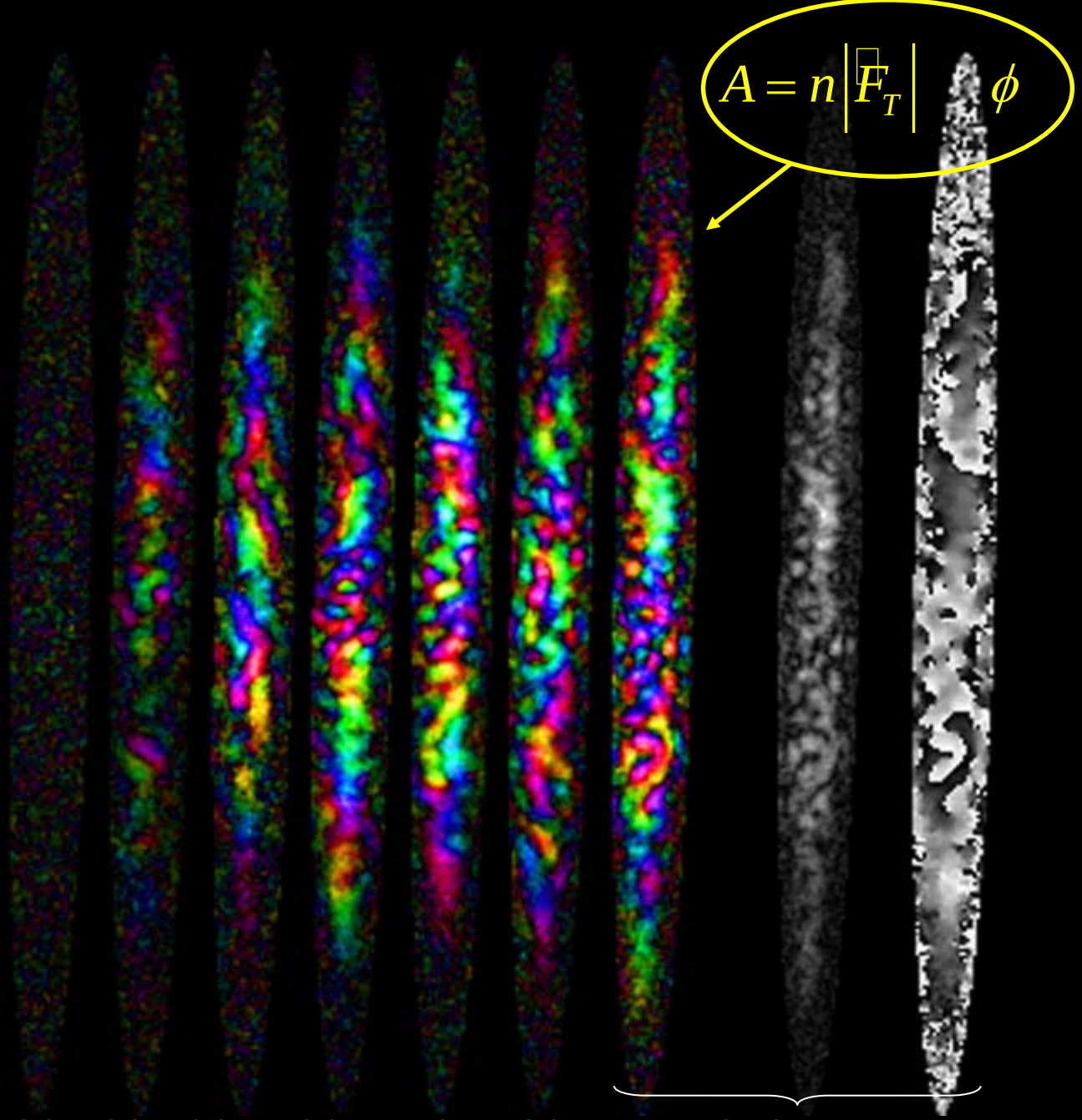


Non-equilibrium (quantum) dynamics at a (quantum) phase transition


B
z
x
probe
-y
T_hold = 30 ms
T_hold = 90 ms
Signal
∝ F_y
0.4
0.2
0.0
-0.2
-0.4
0
5
10
15
20
Point number
nF_T = n(F_x + iF_y) ∝ Ae^{iφ}
C × Asin(ωt + φ)]
330 μm
T_hold = 150 ms
T_hold = 210 ms

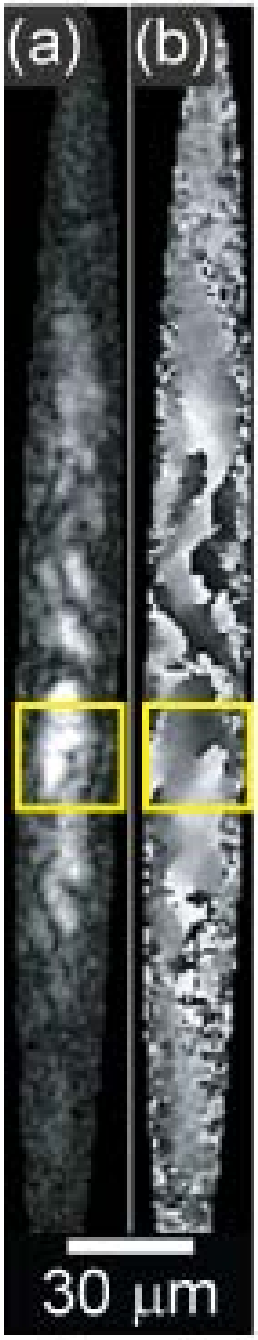
Spontaneously formed ferromagnetism

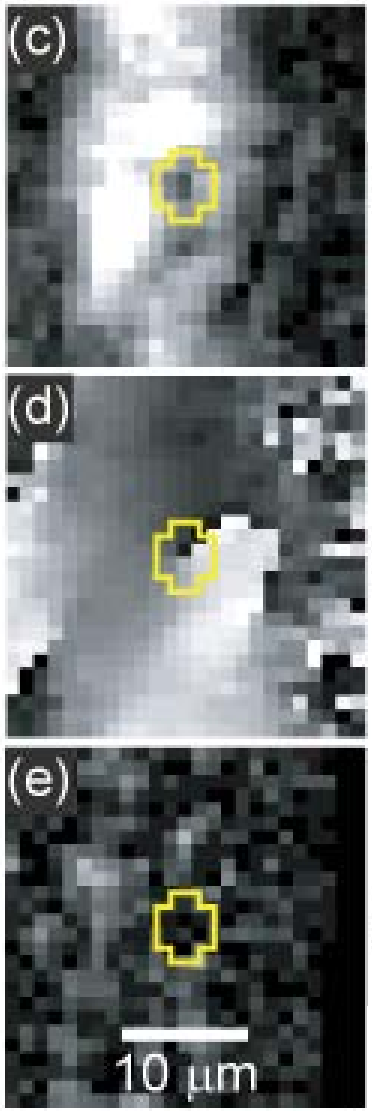
- inhomogeneously broken symmetry
- ferromagnetic domains, large and small
- unmagnetized domain walls marking rapid reorientation



1. **Some interesting phenomena in multi-component Bose gases**: Fragmentation, symmetry breaking, magnetism
2. **Spinor gas**: Definition, symmetries and interactions, mean-field and many-body ground states
3. **Experimental realities**: spin conservation, first studies
4. **Detecting internal-state coherence optically**
5. **Spin dynamics**: oscillations, quantum quench, dynamical instability
6. **Future directions**

Spontaneously formed spin vortices





$|F_T|$

A

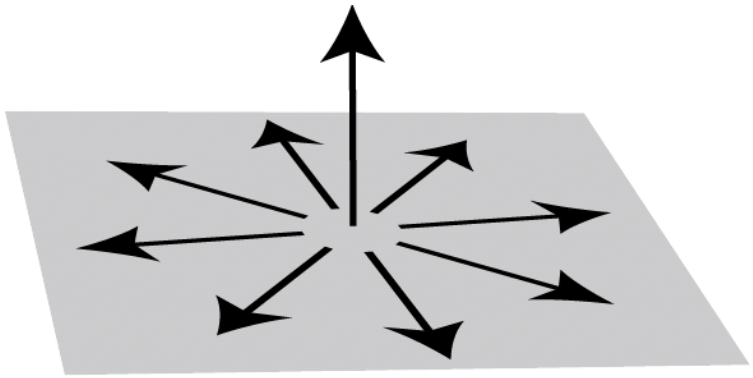
ϕ

A_L

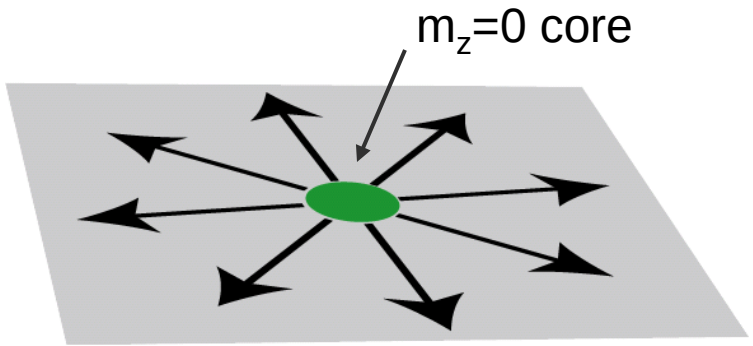
$|F_L|$

T_{hold} = 150 ms

candidates:



Mermin-Ho vortex (meron)

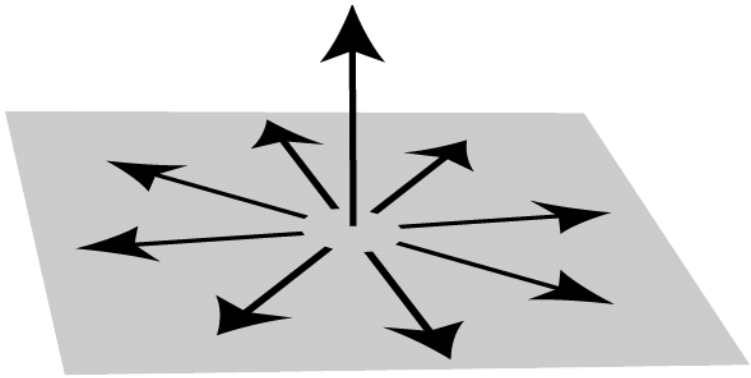


“Polar core” spin vortex

Spontaneously formed spin vortices

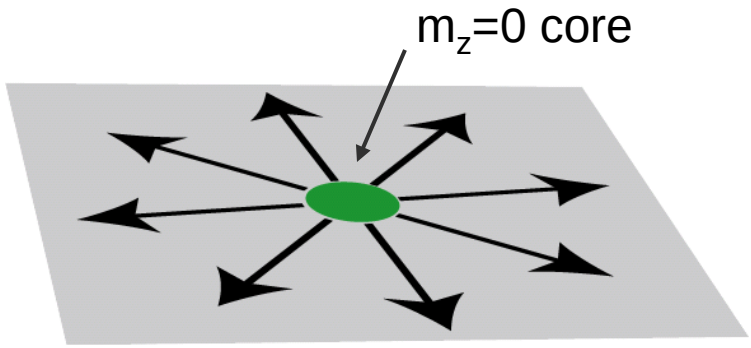
candidates:

$$\vec{\Psi} = \begin{pmatrix} a(r) & \times & 1 \\ b(r) & \times & e^{-i\phi} \\ c(r) & \times & e^{-2i\phi} \end{pmatrix}$$



Mermin-Ho vortex (meron)

$$\vec{\Psi} = \begin{pmatrix} a(r) & \times & e^{i\phi} \\ b(r) & \times & 1 \\ c(r) & \times & e^{-i\phi} \end{pmatrix}$$



“Polar core” spin vortex

Broken chiral symmetry;
Saito, Kawaguchi, Ueda, PRL **96**, 065302 (2006)

More to do

- Quench dynamics
  - Is it quantum noise? Is it a quantum amplification process?
  - measure scaling quantitatively
  - spatial vs. temporal sweeps
- Other spin dynamics
  - spin transport above and below Tc
- Many-body states (sodium?)
- Evolution and role of topological defects
- Dipolar interactions
- Symmetries breaking:
  - Do BEC and ferromagnetism occur at once (same temperature, same timescale, same spatial scale)?
- Applications to magnetometry